

# Corrigendum to “Kernel density estimation for directional-linear data”

Eduardo García-Portugués<sup>1,2,4</sup>, Rosa M. Crujeiras<sup>3</sup>, and Wenceslao González-Manteiga<sup>3</sup>

The proof in Remark 4 of García-Portugués et al. (2013) should be replaced by the following (simpler) one, as the original is based on an erroneous argument. The replacement has no impact in the paper.

**Remark 4.** *The proof of  $\mathbb{E}[|V_n - \mathbb{E}[V_n]|^{2+\delta}] = \mathcal{O}(\mathbb{E}[|V_n|^{2+\delta}])$  is simple. For example, using the  $C_p$  inequality with  $p = 2 + \delta$ :  $|a + b|^{2+\delta} \leq 2^{1+\delta}(|a|^{2+\delta} + |b|^{2+\delta})$ , with  $a, b \in \mathbb{R}$ . Then,*

$$\begin{aligned}\mathbb{E}[|V_n - \mathbb{E}[V_n]|^{2+\delta}] &\leq 2^{1+\delta} \mathbb{E}[|V_n|^{2+\delta} + |\mathbb{E}[V_n]|^{2+\delta}] \\ &= 2^{1+\delta} \left( \mathbb{E}[|V_n|^{2+\delta}] + |\mathbb{E}[V_n]|^{2+\delta} \right) \\ &\leq 2^{2+\delta} \mathbb{E}[|V_n|^{2+\delta}],\end{aligned}$$

where the last step follows by Jensen’s inequality applied to the convex function  $|\cdot|^{2+\delta}$ .

Also, the square integrability of  $f$  (on  $\Omega_q$ ) stated in assumption **D1** is superfluous, as it is implied by the previous conditions stated in **D1**.

## References

García-Portugués, E., Crujeiras, R. M., and González-Manteiga, W. (2013). Kernel density estimation for directional-linear data. *J. Multivar. Anal.*, 121:152–175.

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<sup>1</sup>Department of Statistics, Carlos III University of Madrid (Spain).

<sup>2</sup>UC3M-Santander Big Data Institute, Carlos III University of Madrid (Spain).

<sup>3</sup>Department of Statistics, Mathematical Analysis and Optimization, University of Santiago de Compostela (Spain).

<sup>4</sup>Corresponding author. e-mail: edgarcia@est-econ.uc3m.es.