

The cheap bootstrap

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The bootstrap is a cornerstone of modern statistical inference, valued for its flexibility and data-driven nature. However, in complex or large-scale models its computational demands, stemming from repeated model refitting, can be prohibitive. To address this, Lam (2022) and Lam & Liu (2023) introduced the cheap bootstrap, which reduces the number of bootstrap replications to as few as a single resample while maintaining asymptotic validity. The cheap bootstrap confidence interval takes the form

$$\left[\hat{\psi}_n - t_{B,1-\alpha/2} S_{n,B}, \hat{\psi}_n + t_{B,1-\alpha/2} S_{n,B} \right],$$

where $\hat{\psi}_n$ is the original sample statistic, $S_{n,B}^2 = (1/B) \sum_{b=1}^B (\hat{\psi}_n^{*b} - \hat{\psi}_n)^2$, $t_{B,1-\alpha/2}$ is the $(1 - \alpha/2)$ -quantile of a t -distribution with B degrees of freedom, and $\{\hat{\psi}_n^{*1}, \dots, \hat{\psi}_n^{*B}\}$ is the collection of bootstrap replicates. These works develop asymptotic guarantees for such intervals in both low- and high-dimensional contexts, including linear, logistic, and ridge regression.

This thesis will: (1) review the theoretical underpinnings of the cheap bootstrap, replicating the key proofs; (2) replicate and extend the numerical experiments from the papers, comparing the cheap bootstrap with the conventional bootstrap across estimation problems, and quantifying the coverage of confidence intervals, their length, and the effect of B on precision and computation time; (3) illustrate the use of the cheap bootstrap in large-scale predictive modeling settings where computational resources are a limiting factor.

References

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- Lam, H., & Liu, Z. (2023). Bootstrap in high dimension with low computation. *Proceedings of the 40th International Conference on Machine Learning (ICML), 202*, 18362–18384. <https://proceedings.mlr.press/v202/lam23a.html>