

A correction on “On new omnibus tests of uniformity on the hypersphere”

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We provide a correction to the real data application presented in Section 7 of the original paper. The issue stems from an error in the transformation of the raw data, which affects the correct implementation of the proposed tests. In addition, following the biologically motivated analysis in Stirling et al. (2024), two observations from the summer season are removed from the dataset. In their work, the analysis was carried out using solar times, after applying a suitable transformation to account for solar light. However, we perform the analysis using the original clock times.

As a consequence of these changes, the corrected version of Figure 3 in the original paper is presented here as Figure 1, and the corrected version of Table 5 is reported as Table 1. In addition, the specific quantities in the description of the data (second paragraph in Section 7) should be replaced by the following:

- 1.2 “227 individual observations” should be “225 individual observations”.
- 1.4 “158 in summer” should be “156 in summer”.

Although the methodological conclusions of the original paper remain unchanged, the interpretation of the polar bear data requires slight revision. In particular, the last paragraph of Section 7 should be replaced by the following:

“Table 1 shows that uniformity is not rejected at the significance level $\alpha = 0.05$ for the tests based on $T_{n,1}$ and $T_{n,2}$, neither for specific values of κ and ρ , nor for the 10-fold version, across all periods considered. A similar behavior is observed for the classical uniformity tests, which do not reject uniformity for either season or for the full field season. Overall, the results

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reinforce the conclusion that the observed nursing times are compatible with uniformity on the circle.”

Additionally, we noticed some minor typos in the paper and the supplementary materials (SM). The corrections are provided in the following list:

- p.1511, eq.(2): It should read $c_{k,q} := \frac{\omega_q}{\omega_{q-1}} \left(1 + \frac{2k}{q-1}\right)^{-2} d_{k,q}$.
- p.1513, l.6: (within Section 2.3) Add the sentence “A possible numerical stabilization of the kernel is $(1 - \rho)^{(q+1)}/(1 - \rho^2)\psi_2(\theta; \rho)$ ” after “distribution).”.
- SM, p.8, eq.(25): It should be $\mathbf{x} \mapsto \frac{c_{q,\kappa_{\text{dev}},f}}{\omega_{q-1}} f(\mathbf{x}'\boldsymbol{\theta}; \kappa_{\text{dev}})$.
- SM, p.8, l.10: Add the sentence “Note $\omega_0 = 2$ when $q = 1$.” between “... constant.” and “We consider:”
- SM, p.8, eq.(26): It should be $\mathbf{x} \mapsto \frac{c_{q,\kappa_{\text{dev}},f}}{2(q+1)\omega_{q-1}} \sum_{k=1}^{2(q+1)} f((-1)^r \mathbf{x}'\mathbf{e}_r; \kappa_{\text{dev}})$.

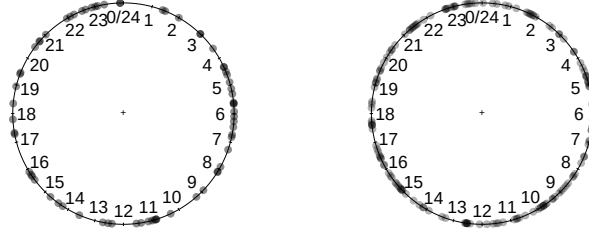


Figure 1: Starting times of nursing periods observed on wild polar bears of the Canadian Arctic during spring (left) and summer (right) of the time span 1973–1999.

Season	n	$T_{n,1}(\kappa)$			$T_{n,1}^{(10)}$	$T_{n,2}(\rho)$			$T_{n,2}^{(10)}$	R_n	B_n	PAD	F_n
		0.1	1	10		0.25	0.5	0.75					
Spring	69	0.49	0.56	0.42	0.83	0.54	0.52	0.48	0.78	0.48	0.76	0.52	0.56
Summer	156	0.68	0.79	0.90	0.93	0.81	0.88	0.90	0.73	0.67	0.76	0.84	0.85
Field	225	0.95	0.96	0.84	0.96	0.96	0.93	0.85	0.86	0.94	0.67	0.93	0.93

Table 1: p -values of several uniformity tests applied to the starting times of nursing periods observed in wild polar bears. The p -values are obtained from the $T_{n,\ell}$ tests for specific parameters, the 10-fold tests $T_{n,\ell}^{(10)}$, and other available tests of uniformity. The p -values were obtained through Monte Carlo ($M = 10^4$).

References

- Stirling, I., Burns, L. E., Regehr, E. V., Laidre, K. L., & Spencer, C. (2024). Nursing behavior of wild polar bears in the Canadian High Arctic. *Canadian Journal of Zoology*, 102, 663–672. doi:<https://doi.org/10.1139/cjz-2024-0001>.