Nonparametric estimation of the spherical regression function with spatially correlated errors

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When data present certain complexities, classical regression procedures designed for Euclidean data can not be directly employed and have to be conveniently modified to provide useful results. This is the case, for instance, when working with directional data, whose support is the hypersphere $\mathbb{S}^d = \{\boldsymbol{x} \in \mathbb{R}^{d+1} : \|\boldsymbol{x}\| = 1\}$ of arbitrary dimension $d \geq 1$ (see Mardia & Jupp (1999)). Moreover, it may occur that the data of interest can present a certain type of dependence. For example, they can be spatially correlated, where observations which are close in space tend to be more similar than observations that are far apart (see Cressie (1993)), thus breaking the omnipresent iid framework.

For a regression model with response $\mathbb{S}^1$ and an $\mathbb{R}^d$-valued covariate, in the presence of (unknown) spatial correlation, local polynomial regression estimators were introduced and analyzed in Meilán-Vila et al. (2021). The goal of this thesis proposal is to extend the existing nonparametric methodology to estimate the regression function to the situation where the response is $\mathbb{S}^2$, and particularly working on data-driven ways of selecting the smoothing parameter. This will require working with a particular correlation structure for the $\mathbb{S}^2$-valued errors (Bevilacqua et al., 2020; Cao et al., 2023). The proposed methodology will be studied via simulations and applied in a real case study, potentially in astronomy.

References

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